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Spin transport properties of a quantum dot coupled to ferromagnetic leads with noncollinear magnetizations

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Abstract
A correct general formula for the spin current through an interacting quantum dot coupled to ferromagnetic leads with magnetization at an arbitrary angle θ is derived within the framework of the Keldysh formalism. Under asymmetric conditions, the spin current component J_z may change sign for $0 < \theta < \pi$. It is shown that the spin current and spin tunneling magnetoresistance exhibit different angle dependence in the free and Coulomb blockade regimes. In the latter case, the competition of the spin precession and the spin-valve effect could lead to an anomaly in the angle dependence of the spin current.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the new field of spintronics [1], the magnetic properties of quantum devices control the transport properties via electron spin, for example, the tunnel magnetoresistance (TMR) in ferromagnetic tunnel junctions. The high magnetoresistance in a TMR device is due to the spin-valve effect, namely, the resistance strongly depends on whether the magnetizations of the two ferromagnetic electrodes are parallel or antiparallel. By switching the magnetization of one electrode with respect to the other, the charge current is modulated by the relative angle θ of the two magnetic moments. With the magnetic tunneling injection technique, a pure spin current can be generated and detected experimentally [2, 3]. This substantial progress in experiment makes it feasible to investigate the spin transport properties in mesoscopic systems.

To study the spin-dependent transport properties, a device setup of a quantum dot (QD) coupled to ferromagnetic leads has been proposed [4]. In such a geometry, the charge current can be spin polarized and can induce a net spin current. However, up to now, most of the previous works have been devoted to charge transport properties, not to the study of the spin current itself. Moreover, the main focus was on the charge transport on a QD coupled to two ferromagnetic leads with collinear magnetizations [5–16], while less attention was given to the noncollinear alignment [17–27]. Braun *et al* [28] gave an expression for the spin current through the left tunnel barrier. However, they did not derive an adequate unified formula for the spin current through the two tunnel barriers and did not actually consider the spin current in a steady state. The z-component of the spin current defined as a difference between the spin-up and spin-down contributions to the charge current was considered by Mu *et al* [29] for the noncollinear case. Unfortunately, these authors did not properly take into account the difference of the spin quantization axis for the two leads, so their result is correct only for the parallel case.

Recently, Rudziński *et al* [20] studied the charge current through a quantum dot coupled to noncollinearly polarized ferromagnetic leads. They found that the current–voltage curve reveals typical step-like characteristics. They also found that the spin precession is enhanced by the Coulomb correlations and strong spin polarization of the leads. Moreover, the relationship between the charge current and the angle of the magnetization configurations of the electrodes has been studied by Zhou *et al* [26]. These authors concluded that the angle dependence of the electric current in the free regime varies monotonically from the parallel to antiparallel alignment, while in the Coulomb blockade regime it varies nonmonotonically. However, authors of both references did not consider the spin current in this general configuration.

In this paper, we first derive an exact general formula for the spin current through a QD coupled to noncollinear ferromagnetic leads, starting from the Heisenberg equation for the spin operator in terms of the Keldysh Green's functions [28] (section 2). To the best of our knowledge, this general formula of the spin current for this class of devices is derived for the first time. It should play a similar role to its charge counterpart derived earlier [4, 30, 31]. Then, by using the equation-of-motion technique with the Hartree–Fock decoupling scheme, the spin current is obtained as a function of the bias voltage and the angle θ of the magnetization configurations of the leads (section 3). Furthermore, the spin current and the spin tunneling magnetoresistance (STMR) are calculated numerically in both free and Coulomb blockade regimes (section 4). The interplay of the spin precession enhanced by the Coulomb repulsion and the spin-valve effect gives rise to anomalous behavior of the angular dependence of the spin current anticipated in the Coulomb blockade regime.

2. General expression for the spin current

The system considered in this paper is schematically shown in figure 1, and it consists of a single-level quantum dot coupled to two ferromagnetic metallic leads by tunneling barriers. The magnetic moment M of the left electrode is pointing to the z-direction, while the moment of the right electrode is at an angle θ to the z axis in the x–z plane. We will use the local and global quantization axes to describe the electron spin. The local quantization axes are determined by the local spin polarization in the leads, while the global axes are the local basis in the left electrode. The corresponding model Hamiltonian is given by [4]

$$
\begin{aligned}
H = &\sum_{\mathbf{k},\sigma;\alpha=\mathrm{L,R}} \epsilon_{\mathbf{k},\sigma,\alpha} c^\dagger_{\mathbf{k},\sigma,\alpha} c_{\mathbf{k},\sigma,\alpha} + \sum_{\gamma} \epsilon_d d^\dagger_\gamma d_\gamma + U d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow \\
&+ \sum_{\mathbf{k}} \left[T_{\mathbf{k},\mathrm{L}} \left(c^\dagger_{\mathbf{k},+,\mathrm{L}} d_\uparrow + c^\dagger_{\mathbf{k},-,\mathrm{L}} d_\downarrow \right) + \text{h.c.} \right] \\
&+ \sum_{\mathbf{k}} \Bigg[T_{\mathbf{k},\mathrm{R}} \left(c^\dagger_{\mathbf{k},+,\mathrm{R}} \cos\frac{\theta}{2} - c^\dagger_{\mathbf{k},-,\mathrm{R}} \sin\frac{\theta}{2} \right) d_\uparrow \\
&+ T_{\mathbf{k},\mathrm{R}} \left(c^\dagger_{\mathbf{k},-,\mathrm{R}} \cos\frac{\theta}{2} + c^\dagger_{\mathbf{k},+,\mathrm{R}} \sin\frac{\theta}{2} \right) d_\downarrow + \text{h.c.} \Bigg], \qquad (1)
\end{aligned}
$$

where the spin projection on the local axes is denoted as $\sigma = \pm$, $\epsilon_{\mathbf{k},\sigma,\alpha} = \epsilon_{\mathbf{k},\alpha} + \sigma M_\alpha$ is the single-electron energy in the αth electrode, and $c^\dagger_{\mathbf{k},\sigma,\alpha}$ and $c_{\mathbf{k},\sigma,\alpha}$ correspond to the creation and annihilation operators, respectively. Similarly, the spin projection on the global axes is denoted as $\gamma = \uparrow\downarrow$, $d^\dagger_\gamma$ and d_γ are the creation and annihilation operators of the electron on the quantum dot with energy ϵ_d.

For simplicity, we can rewrite the model Hamiltonian into a compact matrix form

$$
\begin{aligned}
H = &\sum_{\mathbf{k},\alpha=\mathrm{L,R}} \hat{\mathbf{C}}^\dagger_{\mathbf{k},\alpha} \hat{\epsilon}_{\mathbf{k},\alpha} \hat{\mathbf{C}}_{\mathbf{k},\alpha} + \hat{\epsilon}_d \hat{\mathbf{D}}^\dagger \hat{\mathbf{D}} \\
&+ \frac{U}{4} \left[(\hat{\mathbf{D}}^\dagger \hat{\mathbf{D}})^2 - (\hat{\mathbf{D}}^\dagger \hat{\sigma}_z \hat{\mathbf{D}})^2 \right] \\
&+ \sum_{\mathbf{k},\alpha=\mathrm{L,R}} \left(\hat{\mathbf{C}}^\dagger_{\mathbf{k},\alpha} \hat{\mathbf{T}}_{\mathbf{k},\alpha} \mathbf{R}^\dagger_\alpha \hat{\mathbf{D}} + \text{h.c.} \right), \qquad (2)
\end{aligned}
$$

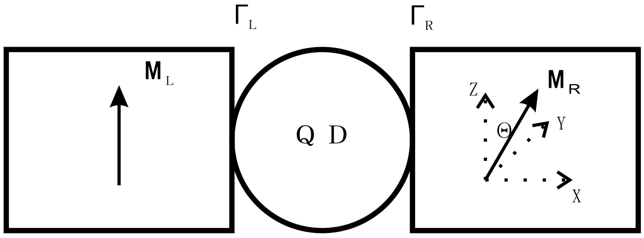

Figure 1. Sketch of the system configuration. QD is coupled to two ferromagnetic leads with magnetizations M_L and M_R at an angle θ.

where we have introduced the Nambu spinors and two useful matrices

$$
\hat{\mathbf{C}}_{\mathbf{k},\alpha} = \begin{pmatrix} C_{\mathbf{k},+,\alpha} \\ C_{\mathbf{k},-,\alpha} \end{pmatrix}, \qquad \hat{\mathbf{D}} = \begin{pmatrix} d_\uparrow \\ d_\downarrow \end{pmatrix},
$$
$$
\hat{\epsilon}_{\mathbf{k},\alpha} = \begin{pmatrix} \epsilon_{\mathbf{k},+,\alpha} & 0 \\ 0 & \epsilon_{\mathbf{k},-,\alpha} \end{pmatrix}, \qquad \mathbf{R}_\alpha = \begin{pmatrix} \cos\frac{\theta_\alpha}{2} & -\sin\frac{\theta_\alpha}{2} \\ \sin\frac{\theta_\alpha}{2} & \cos\frac{\theta_\alpha}{2} \end{pmatrix}, \qquad (3)
$$

with $\theta_\mathrm{L} = 0$ for the left lead and $\theta_\mathrm{R} = \theta$ for the right lead. When the spin operators of the two leads are considered $\hat{\mathbf{S}}_\alpha = (\hbar/2) \sum_{\mathbf{k}} \hat{\mathbf{C}}^\dagger_{\mathbf{k},\alpha} \hat{\sigma}_\alpha \hat{\mathbf{C}}_{\mathbf{k},\alpha}$, the spin matrices are given by

$$
\begin{aligned}
&\sigma^x_\mathrm{L} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^y_\mathrm{L} = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \\
&\sigma^z_\mathrm{L} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \sigma^x_\mathrm{R} = \begin{pmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix}, \qquad (4) \\
&\sigma^y_\mathrm{R} = \sigma^y_\mathrm{L}, \; \sigma^z_\mathrm{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}.
\end{aligned}
$$

From the Heisenberg equation, we can calculate the spin current $\mathbf{J}_\alpha = \langle \hat{\mathbf{J}}_\alpha \rangle$ from the lead to the dot [28]

$$
\begin{aligned}
\hat{\mathbf{J}}_\alpha &= \frac{\mathrm{i}}{\hbar} [\hat{\mathbf{S}}_\alpha, H] \\
&= \frac{\mathrm{i}}{2} \sum_{\mathbf{k}} \mathrm{Tr}\left(\hat{\mathbf{C}}^\dagger_{\mathbf{k},\alpha} \hat{\sigma}_\alpha \hat{\mathbf{T}}_{\mathbf{k},\alpha} \mathbf{R}^\dagger_\alpha \hat{\mathbf{D}} - \hat{\mathbf{D}}^\dagger \mathbf{R}_\alpha \hat{\mathbf{T}}^*_{\mathbf{k},\alpha} \hat{\sigma}_\alpha \hat{\mathbf{C}}_{\mathbf{k},\alpha} \right). \qquad (5)
\end{aligned}
$$

Moreover, by introducing the Keldysh Green's function matrices

$$
\begin{aligned}
\hat{\mathbf{G}}^<_\alpha(\mathbf{k}, t) &= \mathrm{i} \begin{pmatrix} \langle C^\dagger_{\mathbf{k},+,\alpha}(0) d_\uparrow(t) \rangle & \langle C^\dagger_{\mathbf{k},-,\alpha}(0) d_\uparrow(t) \rangle \\ \langle C^\dagger_{\mathbf{k},+,\alpha}(0) d_\downarrow(t) \rangle & \langle C^\dagger_{\mathbf{k},-,\alpha}(0) d_\downarrow(t) \rangle \end{pmatrix}, \\
\hat{\mathbf{G}}^<_d(t) &= \mathrm{i} \begin{pmatrix} \langle d^\dagger_\uparrow(0) d_\uparrow(t) \rangle & \langle d^\dagger_\downarrow(0) d_\uparrow(t) \rangle \\ \langle d^\dagger_\uparrow(0) d_\downarrow(t) \rangle & \langle d^\dagger_\downarrow(0) d_\downarrow(t) \rangle \end{pmatrix}, \qquad (6)
\end{aligned}
$$

we can further rewrite the expectation value of the spin current as

$$
\mathbf{J}_\alpha = \sum_{\mathbf{k}} \int \frac{\mathrm{d}\omega}{2\pi} \mathrm{Re}\left[\mathrm{Tr}\left(\hat{\mathbf{G}}^<_\alpha(\mathbf{k}, \omega) \hat{\sigma}_\alpha \hat{\mathbf{T}}_{\mathbf{k},\alpha} \mathbf{R}^\dagger_\alpha \right) \right], \qquad (7)
$$

where $\hat{\mathbf{G}}^<_\alpha(\mathbf{k}, \omega)$ is the Fourier transform of $\hat{\mathbf{G}}^<_\alpha(\mathbf{k}, t)$. Since the ferromagnetic leads are noninteracting, we obtain the Dyson equation for $\hat{\mathbf{G}}^<_\alpha(\mathbf{k}, \omega)$ in terms of the Green's function matrices for the local dot electrons,

$$
\hat{\mathbf{G}}^<_\alpha(\mathbf{k}, \omega) = \hat{\mathbf{G}}^r_d(\omega) \mathbf{R}_\alpha \hat{\mathbf{T}}^*_{\mathbf{k},\alpha} \hat{\mathbf{g}}^<_\alpha(\mathbf{k}, \omega) + \hat{\mathbf{G}}^<_d(\omega) \mathbf{R}_\alpha \hat{\mathbf{T}}^*_{\mathbf{k},\alpha} \hat{\mathbf{g}}^a_\alpha(\mathbf{k}, \omega), \qquad (8)
$$

where

$$\hat{\mathbf{g}}^{<}_{\alpha}(\mathbf{k},\omega) = 2\pi \mathrm{i} f_\alpha(\omega) \begin{pmatrix} \delta(\omega-\epsilon_{\mathbf{k},+,\alpha}) & 0 \\ 0 & \delta(\omega-\epsilon_{\mathbf{k},-,\alpha}) \end{pmatrix},$$

$$\hat{\mathbf{g}}^{a}_{\alpha}(\mathbf{k},\omega) = \begin{pmatrix} \frac{1}{\omega-\epsilon_{\mathbf{k},+,\alpha}-\mathrm{i}0^+} & 0 \\ 0 & \frac{1}{\omega-\epsilon_{\mathbf{k},-,\alpha}-\mathrm{i}0^+} \end{pmatrix},$$

with $f_\alpha(\omega) = [1+\exp(\omega-\mu_\alpha)/(k_B T)]^{-1}$, $\mu_L = -eV/2$ and $\mu_R = eV/2$. Inserting these expressions into the spin current formula, we obtain the spin current as follows:

$$\mathbf{J}_\alpha = \int \frac{\mathrm{d}\omega}{4\pi} \mathrm{Re}\Bigg(\mathrm{Tr}\Bigg\{\mathrm{i}\tilde{\mathbf{\Gamma}}_\alpha(\omega)\Bigg[2f_\alpha(\omega)\hat{\mathbf{G}}^r_d(\omega) + \hat{\mathbf{G}}^<_d(\omega) - \mathrm{i}\mathcal{P}\int \frac{\mathrm{d}E}{\pi}\frac{\hat{\mathbf{G}}^<_d(E)}{E-\omega}\Bigg]\Bigg\}\Bigg), \tag{9}$$

where the integral is taken as the principle value and

$$\tilde{\mathbf{\Gamma}}_\alpha(\omega) = \mathbf{R}_\alpha \begin{pmatrix} \Gamma_{+,\alpha}(\omega) & 0 \\ 0 & \Gamma_{-,\alpha}(\omega) \end{pmatrix} \hat{\sigma}_\alpha \mathbf{R}^\dagger_\alpha,\ \Gamma_{\sigma,\alpha}(\omega) = 2\pi \sum_{\mathbf{k}} \left|T_{\mathbf{k},\alpha}\right|^2 \delta(\omega-\epsilon_{\mathbf{k},\sigma,\alpha}). \tag{10}$$

Since this system is quasi one-dimensional, different from the spin Hall systems [32–34] in which the spin–orbit coupling is essential, we do not take into account those spin flip processes due to the spin–orbit coupling. So we do consider the spin current through a QD as a continuous and conserved quantity. The steady state is realized in the system through the scattering process, which is similar to the charge transport. As far as we understand, no one has studied the detailed relaxation process within the QD. In a steady state, the spin current is uniform, so $\mathbf{J}_L = -\mathbf{J}_R$. Thus, we can symmetrize the spin current as $\mathbf{J} = (\mathbf{J}_L - \mathbf{J}_R)/2$ which is similar to the operation performed on the expression for the charge current [4, 30, 31]. The general expression for the spin current is then given by

$$\mathbf{J} = \frac{1}{2}\int \frac{\mathrm{d}\omega}{2\pi}\mathrm{Re}\Bigg\{\mathrm{Tr}\Bigg[\mathrm{i}\left[f_L(\omega)\tilde{\mathbf{\Gamma}}_L(\omega) - f_R(\omega)\tilde{\mathbf{\Gamma}}_R(\omega)\right]\hat{\mathbf{G}}^r_d(\omega) + \left[\tilde{\mathbf{\Gamma}}_L(\omega) - \tilde{\mathbf{\Gamma}}_R(\omega)\right]\left(\frac{\mathrm{i}}{2}\hat{\mathbf{G}}^<_d(\omega) + \mathcal{P}\int\frac{\mathrm{d}E}{2\pi}\frac{\hat{\mathbf{G}}^<_d(E)}{E-\omega}\right)\Bigg]\Bigg\}. \tag{11}$$

Braun *et al* [28] gave an expression for the spin current through the left tunnel barrier, but they did not derive a unified formula for the spin current through the left and right tunnel barriers. Also, these authors did not provide a symmetrized formula in the steady state, which is essential for the calculation and discussion of the spin current. Mu *et al* [29] used the difference between the charge currents through the spin-up and spin-down channels to define the z-component of the spin current. However, these authors did not properly take into account the difference of the two local quantization axes of the two ferromagnetic leads which strongly affects the tunneling Hamiltonian, as pointed out by Rudziński *et al* [20]. Moreover, their expression of the charge current was not symmetrized. As a result, their formula is correct only for the parallel case.

3. Calculation of the Keldysh Green's functions

To investigate the nonequilibrium transport properties, there are two commonly used techniques to calculate the Keldysh Green's functions. One is the real-time diagrammatic technique [9, 19, 22, 35, 36], based on a perturbation expansion in terms of the dot-lead coupling strength, whereas the Coulomb interactions on the dot are exactly taken into account. However, this technique only considers finite-order tunneling processes, and cannot deal with the coupling between the dot and the electrode exactly. The other alternative is the equation-of-motion technique [4, 10, 13, 20, 21, 37] which treats the dot-lead coupling exactly, while the strong correlations on the dot can be dealt with only approximately.

In this paper, the Green's functions are solved by the equation-of-motion technique with the Hartree–Fock decoupling scheme [20, 37]. The solution can be written in a compact form of the matrix Dyson equation

$$\hat{\mathbf{G}}_d(\omega) = \left[\mathbf{1} - \hat{\mathbf{g}}_d(\omega)\mathbf{\Sigma}^{(0)}(\omega)\right]^{-1}\hat{\mathbf{g}}_d(\omega), \tag{12}$$

where

$$\hat{\mathbf{g}}_d(\omega) = \begin{pmatrix} \frac{\omega-\epsilon_d-U(1-\langle n_{\downarrow,\downarrow}\rangle)}{(\omega-\epsilon_d)(\omega-\epsilon_d-U)} & -\frac{U\langle n_{\downarrow,\uparrow}\rangle}{(\omega-\epsilon_d)(\omega-\epsilon_d-U)} \\ -\frac{U\langle n_{\uparrow,\downarrow}\rangle}{(\omega-\epsilon_d)(\omega-\epsilon_d-U)} & \frac{\omega-\epsilon_d-U(1-\langle n_{\uparrow,\uparrow}\rangle)}{(\omega-\epsilon_d)(\omega-\epsilon_d-U)} \end{pmatrix}$$

with $\langle n_{\alpha,\beta}\rangle = \langle d^\dagger_\alpha d_\beta\rangle$ and $\hat{\mathbf{g}}_d(\omega)$ denotes the corresponding Green functions in the matrix form of the uncoupled dot. The self-energy $\mathbf{\Sigma}^{(0)}(\omega)$ is given by

$$\mathbf{\Sigma}^{(0)}(\omega) = \begin{pmatrix} \Sigma^{(0)}_{++}(\omega) & \Sigma^{(0)}_{+-}(\omega) \\ \Sigma^{(0)}_{-+}(\omega) & \Sigma^{(0)}_{--}(\omega) \end{pmatrix},$$

with

$$\begin{aligned} \Sigma^{(0)}_{\pm\pm}(\omega) &= \sum_{\mathbf{k}}\left[\frac{|T_{\mathbf{k},L}|^2}{\omega-\epsilon_{\mathbf{k},\pm,L}} + C_K\left|T_{\mathbf{k},R}\right|^2\right], \\ \Sigma^{(0)}_{\pm\mp}(\omega) &= \tfrac{1}{2}\sum_{\mathbf{k}}\left|T_{\mathbf{k},R}\right|^2 D_K \sin\theta, \\ C_K &= \frac{\cos^2(\theta/2)}{\omega-\epsilon_{\mathbf{k},\pm,R}} + \frac{\sin^2(\theta/2)}{\omega-\epsilon_{\mathbf{k},\mp,R}}, \\ D_K &= \frac{1}{\omega-\epsilon_{\mathbf{k},+,R}} - \frac{1}{\omega-\epsilon_{\mathbf{k},-,R}}. \end{aligned} \tag{13}$$

Then one can calculate the retarded Green functions as

$$G^r_{\uparrow\uparrow}(\omega) = \left[g^r_{\uparrow\uparrow}(\omega) - A(\omega)\Sigma^{(0)r}_{--}(\omega)\right]\Big/B(\omega),$$

$$G^r_{\uparrow\downarrow}(\omega) = \left[g^r_{\uparrow\downarrow}(\omega) - A(\omega)\Sigma^{(0)r}_{+-}(\omega)\right]\Big/B(\omega),$$

$$G^r_{\downarrow\uparrow}(\omega) = \left[g^r_{\downarrow\uparrow}(\omega) - A(\omega)\Sigma^{(0)r}_{-+}(\omega)\right]\Big/B(\omega),$$

$$G^r_{\downarrow\downarrow}(\omega) = \left[g^r_{\downarrow\downarrow}(\omega) - A(\omega)\Sigma^{(0)r}_{++}(\omega)\right]\Big/B(\omega),$$

where

$$A(\omega) = g^r_{\uparrow\uparrow}(\omega)g^r_{\downarrow\downarrow}(\omega) - g^r_{\uparrow\downarrow}(\omega)\,g^r_{\downarrow\uparrow}(\omega),$$

$$B(\omega) = 1 - g^r_{\uparrow\uparrow}(\omega)\Sigma^{(0)r}_{++}(\omega) - g^r_{\downarrow\downarrow}(\omega)\Sigma^{(0)r}_{--}(\omega) - g^r_{\uparrow\downarrow}(\omega)\Sigma^{(0)r}_{-+}(\omega) - g^r_{\downarrow\uparrow}(\omega)\,\Sigma^{(0)r}_{+-}(\omega) + A(\omega)\left[\Sigma^{(0)r}_{++}(\omega)\Sigma^{(0)r}_{--}(\omega) - \Sigma^{(0)r}_{+-}(\omega)\Sigma^{(0)r}_{-+}(\omega)\right].$$

The retarded self-energies $\Sigma^{(0)r}_{\pm\pm}(\omega)$ and $\Sigma^{(0)r}_{\pm\mp}(\omega)$ are given by the formulae

$$\Sigma^{(0)r}_{\pm\pm}(\omega) = -\frac{\mathrm{i}}{2}[\Gamma_{\pm,\mathrm{L}}(\omega) + \Gamma_{\pm,\mathrm{R}}(\omega)\cos^2(\theta/2) + \Gamma_{\mp,\mathrm{R}}(\omega)\sin^2(\theta/2)], \tag{14}$$

$$\Sigma^{(0)r}_{\pm\mp}(\omega) = -\frac{\mathrm{i}}{4}[\Gamma_{+,\mathrm{R}}(\omega) - \Gamma_{-,\mathrm{R}}(\omega)]\sin\theta.$$

In the following we assume

$$\Gamma_{\pm,\mathrm{L}}(\omega) = \Gamma_{\pm,\mathrm{L}} = \Gamma_0(1 \pm p_\mathrm{l}),$$

$$\Gamma_{\pm,\mathrm{R}}(\omega) = \Gamma_{\pm,\mathrm{R}} = \gamma\Gamma_0(1 \pm p_\mathrm{r}),$$

where p_l and p_r denote the spin polarization of the left and right electrodes, respectively, and the parameter γ expresses the asymmetry coupling of the left and right electrodes to the dot. $\hat{\mathbf{G}}^<_d(\omega)$ can be obtained from the Keldysh equation,

$$\hat{\mathbf{G}}^<_d(\omega) = \hat{\mathbf{G}}^r_d(\omega)\mathbf{\Sigma}^<(\omega)\hat{\mathbf{G}}^a_d(\omega), \tag{15}$$

where the full self-energy $\mathbf{\Sigma}^<(\omega)$ is related to $\mathbf{\Sigma}^{(0)<}(\omega)$ via the Ng ansatz [38]

$$\Sigma^<_{\pm\pm}(\omega) = \mathrm{i}\Gamma_0[f_\mathrm{L}(\omega)(1 \pm p_\mathrm{l}) + \gamma f_\mathrm{R}(\omega)(1 \pm p_\mathrm{r}\cos\theta)],$$
$$\Sigma^<_{\pm\mp}(\omega) = \mathrm{i}\gamma\Gamma_0 f_\mathrm{R}(\omega)p_\mathrm{r}\sin\theta. \tag{16}$$

The statistical averages of $\langle n_{\alpha,\beta}\rangle$ have to be calculated self-consistently in the following way:

$$\langle n_{\sigma,\sigma}\rangle = \mathrm{Im}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}G^<_{\sigma\sigma}(\omega),$$
$$\langle n_{\sigma,\bar{\sigma}}\rangle = -\mathrm{i}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}G^<_{\bar{\sigma}\sigma}(\omega). \tag{17}$$

This approximate calculation of the Keldysh Green's functions does not take into account the Kondo-like correlations which need a careful treatment of the Coulomb interaction on the dot. Some previous works which studied the charge transport properties of this system discussed the Kondo effect, including the collinear alignment [5–12, 15, 16] and the noncollinear case [4, 21, 23, 27]. It is left for our future work to discuss the influence of the Kondo-like correlations on the spin transport properties of this system.

4. Results and discussions

Now we numerically calculate the three components of the spin current. Since a general magnetic configuration of the leads is considered, the spin tunneling magnetoresistance (STMR) can be estimated by

$$\mathrm{STMR}_a = \frac{J_a(\theta = 0) - J_a(\theta)}{J_a(\theta = 0)}, \qquad a = x, y, z \tag{18}$$

where $J_{x,y,z}(\theta)$ denote the three components of the spin current. In the following three different situations are considered: a symmetric junction with fully polarized external electrodes ($p_\mathrm{l} = p_\mathrm{r} = 1$), with partially polarized external electrodes ($p_\mathrm{l} = p_\mathrm{r} = 0.4$), and an asymmetric junction ($p_\mathrm{l} = 0.4$, $p_\mathrm{r} = 1$).

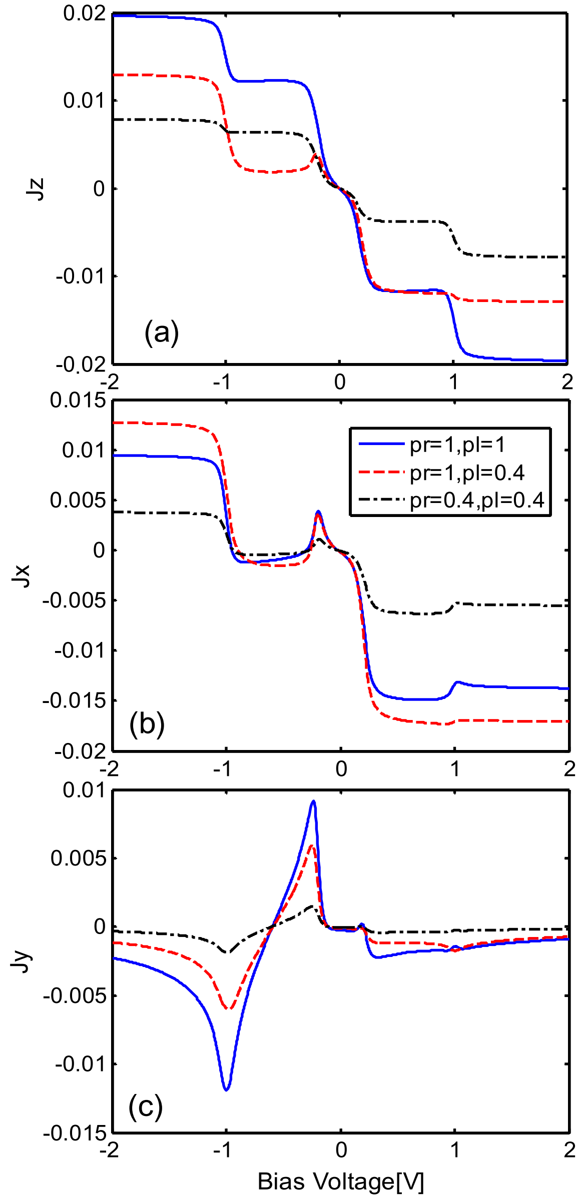

Figure 2. Voltage bias dependence of the spin current for $\theta = \pi/3$. (a) J_z, (b) J_x, and (c) J_y. The parameter values assumed are $\epsilon_d = 0.1$ eV, $U = 0.4$ eV, $\Gamma_0 = 0.01$ eV, $\gamma = 1$, and $T = 100$ K.

The J_z-voltage curve for the symmetric cases reveals typical step-like characteristics. Below the lower threshold voltage, the dot is empty and the sequential contribution to J_z is exponentially suppressed. The first step in J_z occurs at a critical bias, where the discrete level ϵ_d crosses the Fermi level, whereas the step at a higher threshold corresponds to the case when $\epsilon_d + U$ crosses the Fermi level. In the same voltage range, J_z in the case of $p_\mathrm{l} = p_\mathrm{r} = 1$ is much larger, since the external electrodes are fully polarized. Mu *et al* [29] also considered this case ($p_\mathrm{l} = p_\mathrm{r} = 0.4$, $\theta = \pi/3$), but their result is different from ours because they did not properly take into account the difference of the local quantization axes in the two leads and

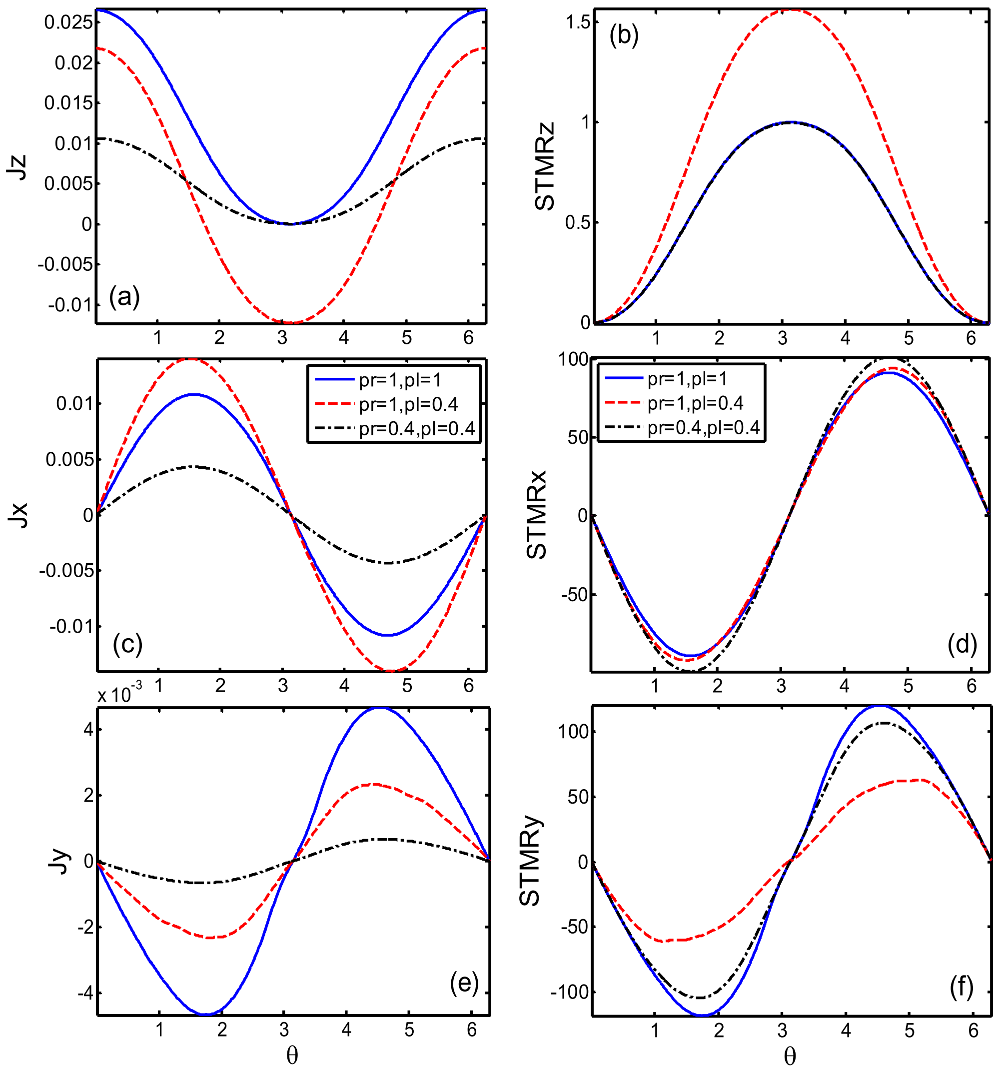

Figure 3. Angle dependence of the spin current and spin tunneling magnetoresistance in the free regime for $v = -1.5$ V. (a) J_z, (b) STMR$_z$, (c) J_x, (d) STMR$_x$, (e) J_y, and (f) STMR$_y$. The parameter values assumed are $\epsilon_d = 0.1$ eV, $U = 0.4$ eV, $\Gamma_0 = 0.01$ eV, $\gamma = 1$, and $T = 100$ K.

hence the result for the z-component of the spin current is incorrect. The case of $p_l = 0.4$, $p_r = 1$ is more complex (red dashed curve in figure 1(a)), as the asymmetry between the left and right electrodes gives rise to asymmetrical transport characteristics of the junction with respect to the bias reversal. For the positive bias, the J_z curve is rather smooth above the first threshold voltage, while for the negative bias, below the first threshold sequential tunneling is exponentially suppressed and only the higher-order tunneling processes are possible. When ϵ_d approaches the Fermi level of the left electrode, resonant tunneling can happen. However, as the bias further increases, J_z is suppressed by an electron on the QD since the electrode is partially polarized (Coulomb blockade effect), and a small peak appears as a result of competition between the resonance tunneling and the Coulomb repulsion. After the second resonant tunneling, J_z finally saturates at a certain level. The behavior of spin current component J_x (figure 2(b)) is similar to the component J_z (figure 2(a)), because the magnetizations of the two leads are aligned in the x–z plane. However, the asymmetry effect resulting in the appearance of a peak at the first threshold is more pronounced. It appears even for the symmetric electrodes ($p_l = p_r = 0.4$, 1), because the Coulomb blockade effect already shows up. The asymmetry of the spin current curve is even more pronounced for the y-component (figure 2(c)). Nevertheless, the two peaks on the J_y curve are exactly located at the two resonant tunneling biases.

The spin current is strongly affected by the angle θ between the magnetic moments of the leads and we can use STMR to describe it. In the free regime, where $|eV/2| > \epsilon_d + U$, the QD energy level may be occupied by two electrons, because the Coulomb correlation plays a small role in the spin tunneling. As a result, J_z and STMR$_z$ exhibit a monotonic variation between the parallel and antiparallel magnetization configurations, which is typical of a normal spin-valve effect. Under the third condition ($p_l = 0.4$, $p_r = 1$), J_z can achieve a negative value. The absolute values of J_x and J_y achieve their maxima between $\theta = 0$ and π, as shown in figures 3(c) and (e), since the absolute values of the x and y components of

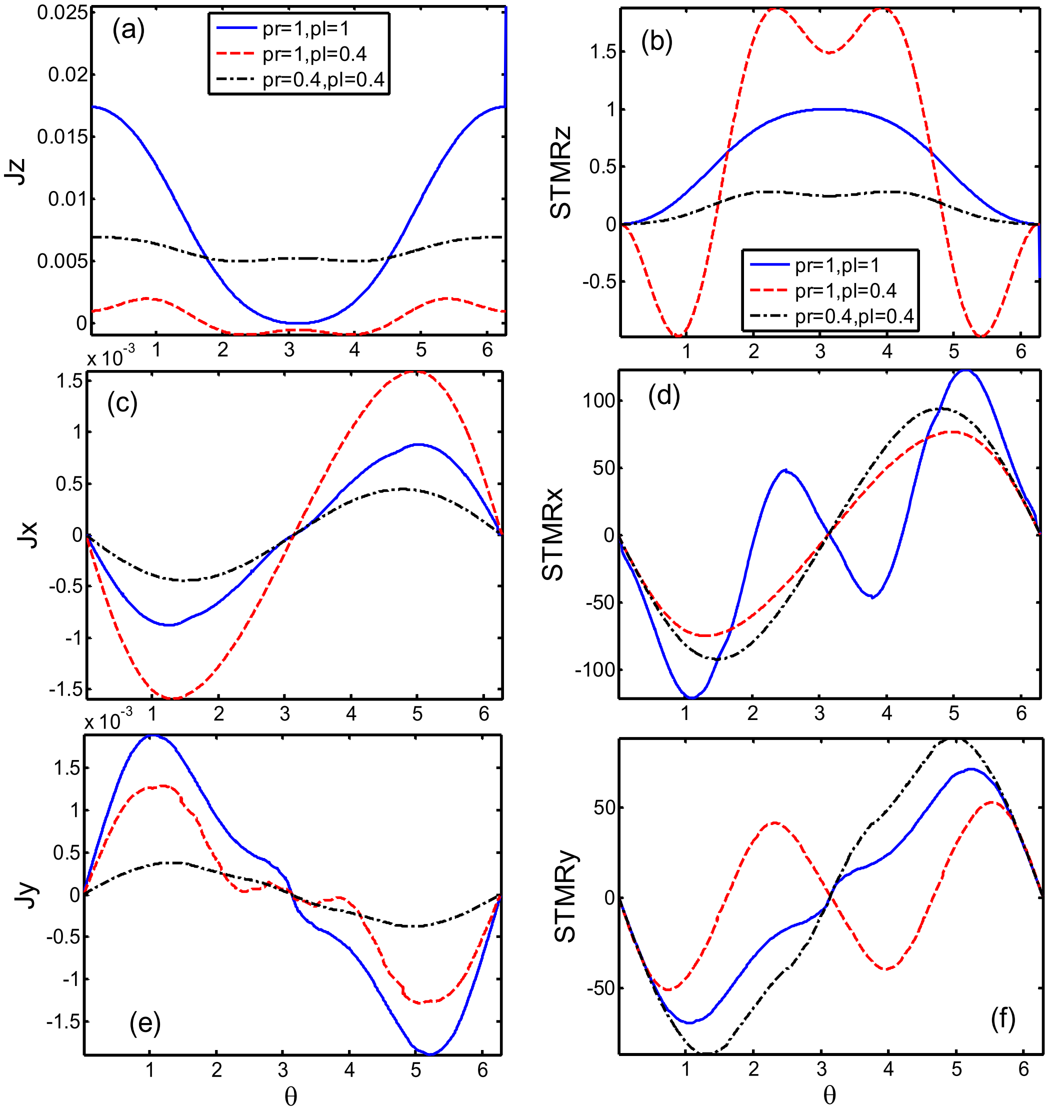

Figure 4. Angle dependence of the spin current and spin tunneling magnetoresistance in the Coulomb blockade regime for $v = -0.5$ V. (a) J_z, (b) STMR_z, (c) J_x, (d) STMR_x, (e) J_y, and (f) STMR_y. The parameters assumed for numerical calculations are $\epsilon_d = 0.1$ eV, $U = 0.4$ eV, $\Gamma_0 = 0.01$ eV, $\gamma = 1$, and $T = 100$ K.

the electron spin in the right electrode may increase when the magnetic moment of the right lead approaches the x–y plane.

In the Coulomb blockade regime $\epsilon_d < |eV/2| < \epsilon_d + U$, the QD energy level can be occupied only by one electron. The Coulomb interaction plays an important role in the spin current through the QD. In figure 4(a), it is found that $J_z(\theta = 0)$ is no longer maximal and $J_z(\theta)$ is greater than $J_z(\theta = 0)$ in a wide range of θ under this condition ($p_l = 0.4$, $p_r = 1$). It is quite different from that in the free regime. The coupling between the QD and the ferromagnetic leads may induce an effective exchange field, and its strength and orientation with respect to the global quantization axis depend on the bias voltage and the angle between magnetizations of the leads. When only one electron resides on the QD energy level, the spin degrees of freedom experience a torque due to the effective exchange field, which results in precession of the spin around the field [17]. This process would suppress J_z, and the competition between the spin precession effect and the spin-valve effect leads to the anomaly of $J_z(\theta)$. As a result of the spin precession, the signs of $J_x(\theta)$ and $J_y(\theta)$ are opposite to those in the free regime.

In conclusion, we have derived a general formula for the spin current through a QD coupled to ferromagnetic leads with noncollinear magnetizations, and used the formula to calculate the spin transport properties of the system. The competition of the spin precession and the spin-valve effect results in an anomaly of the angle dependence of the spin current. Further investigations are needed to carefully treat the Coulomb interaction on the QD.

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